Using "Knowledge of the Crowd" to Inform Subjective and Objective Projectile Point Clustering

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Abstract
Thirty-four volunteers sorted images of 259 projectile points into groups based on their own subjective preferences. Twenty objective parameters consisting of both metrics and proportions were also used to describe the projectile points. By analyzing the subjective groups using the objective parameters, nine of the objective parameters were identified as most related to the characteristics of the points used by each volunteer to make grouping decisions. These nine subjective parameters as well as the twenty objective parameters were then used in three unsupervised clustering algorithms, and the reduction in the standard deviation of the parameters within groups were compared. These were also compared with the reduction in standard deviation of the parameters in the groups created by the volunteers. The clusters using the nine subjective parameters resulted in about twice the reduction in standard deviation compared to those resulting from the subjective grouping alone. This indicates the points in the objective clustering groups were more similar than those grouped subjectively by volunteers. Thus, objective clustering could produce both more repeatable results and groups that contained more morphologically similar points.

Introduction
Differences and similarities between projectile point shapes have been used to create projectile point typologies for cultural and temporal periods within geographic regions (Holmer 1978, Thomas 1981, Phagan 1988, Justice 2002). These projectile point groupings were created utilizing both subjective observations, i.e., perceived similarities and differences by one or more persons (Gunn and Prewitt 1975), and objective methods, i.e., using measurements and mathematical algorithms (Holmer 1980, Phagan 1988). In both approaches, the results are groupings of projectile points that are more similar within a group than to those in another group as defined by the respective methods used.

In the objective case, the membership of the groups may change due to different algorithms as well as the use of different parameters (characteristics) of the projectile points. Similarly, in the subjective case, the membership of groups may be different for groups defined by different people due to differences in the significance awarded to various characteristics of the projectile points.

This investigation used groups created independently by 34 volunteers and groups created by three mathematical clustering algorithms from images of 259 projectile points (examples shown in Figure 1). A variety of volunteers were actively sought to compare and contrast the differences between persons with different levels of experience working with projectile points as well as between subjective (volunteer derived) and objective (mathematically derived) groups. The 259 projectile points consisted of side notched, corner notched, and stemmed points from 5MT6970, a Pueblo II period site in southwestern Colorado, (Bradley 2018, Cross 2015) and 5JF51, an Archaic and Early Ceramic rock shelter site west of Denver, CO (CCR 2021).

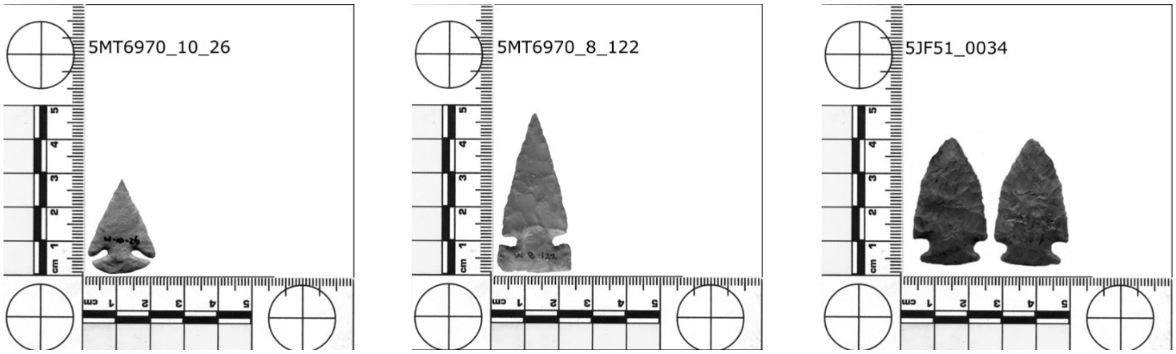

Figure 1. Examples of Images Provided to Volunteers

Approach

<u>Subjective Groupings</u>

Thirty-four volunteers were instructed to sort printed black and white images of 259 projectile points into groups they thought were visually similar. They were told there was no right or wrong number of groups. They were also asked to fill out a short questionnaire to obtain age, gender, occupation, and how much experience they had working with projectile points. The majority of those that responded were older. The level of experience working with projectile points was broken into advanced, intermediate, and novice experience. Advanced were professional, trained archaeologists with experience using existing typologies, intermediate were avocationalists that had some training and some experience classifying projectile points, and novice were those that had no training or experience classifying projectile points. The result was 8 advanced, 12 intermediate and 14 novice responses (Table 1).

Table 1: Respondent Demographics

Age Group (years)	Experience	Male	Female	Totals
20–35	Advanced	1	-	1
	Intermediate	-	-	-
	Novice	-	1	1
36–55	Advanced	2	-	2
	Intermediate	-	-	-
	Novice	-	-	-
56–65	Advanced	-	-	-
	Intermediate	3	3	6
	Novice	3	2	5
>65	Advanced	3	2	5
	Intermediate	5	1	6
	Novice	5	3	8
Totals		**22**	**12**	**34**
Experience	Advanced	Intermediate	Novice	Total
Totals	**8**	**12**	**14**	**34**

<u>Objective Groupings</u>

For the objective, mathematical clustering, measurements were taken from the images on the most complete vertical half of each of the 259 projectile points. Using tpsDig232, a free software tool developed by Stony Brook University for morphometric analysis (Rohlf 2004) eighteen locations on each projectile point, landmarks (LM), were recorded. Two additional locations, 3 cm apart on the embedded scale, were taken to allow scale corrections of each image (Table 2 and Figure 2). Using the coordinates of those landmarks, 20 different measurements and ratios based on modified parameters from Thomas (1981), Berry (2020), and Gunn and Prewitt (1975) were calculated for each of the 259 projectile points (Table 3 and Figure 3).

Table 2: Landmark Definitions

Landmark (LM) No.	Defined location
1	On left blade edge of point close to the top
2	On left blade edge below LM1 at significant change in slope of edge
3	On left blade edge below LM2 at next significant change in slope of edge
4	On left glade edge below LM3 near shoulder, used with LM3 to define blade slope
5	On left side of tang or on the end of tang if tang or shoulder comes to a point
6	On right side of tang or shoulder or on LM5 if tang or shoulder comes to a point
7	On distal edge of notch towards body, used with LM6 to define distal edge angle
8	Back of the notch or intersection of distal and proximal edges
9	On proximal edge closest to LM8, used with LM10 to define proximal edge angle
10	On left end of proximal edge, used with LM10 to define proximal edge angle
11	Furthest left extent of base
12	Location of last change of slope that defines bottom of the base
13	Location of left side of any basal notch or basal concavity
14	Location of highest (or deepest) part of basal notch or concavity
15	Location of lowest point on base
16	Location of center point of bottom of the base
17	Back of notch or intersection of distal and proximal edges on right side
18	At the point tip (or co-located with LM17 if tip was missing)
19	Place on scale in image at 1 cm
20	Place on scale in image at 4 cm

Table 3: Measurements and Ratios (Properties) Used

1	Shoulder Width
2	Neck Width
3	Notch Direction
4	Notch Depth
5	Total Angle
6	Distal Notch Edge Angle
7	Proximal Notch Edge Angle
8	Base Width
9	Maximum Base Extent
10	Base Depth
11	Base Concavity
12	Shoulder Extent
13	Margin Angle
14	Neck Width/Shoulder Width

15	Base Width/Shoulder Width
16	Depth of Base/Base Width
17	Maximum Base Extent /Shoulder Width
18	Depth of Base/Shoulder Width
19	Neck Width/ Base Width
20	Shoulder Extent/Maximum Base Extent

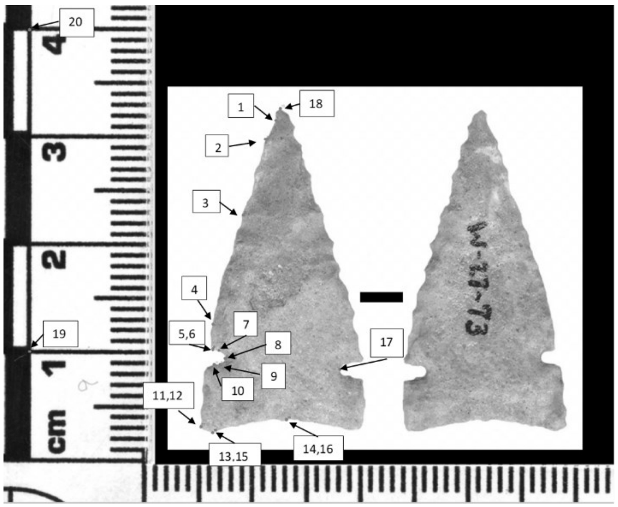

Figure 2: Example of Placement of the 20 Landmarks on Projectile Point Image

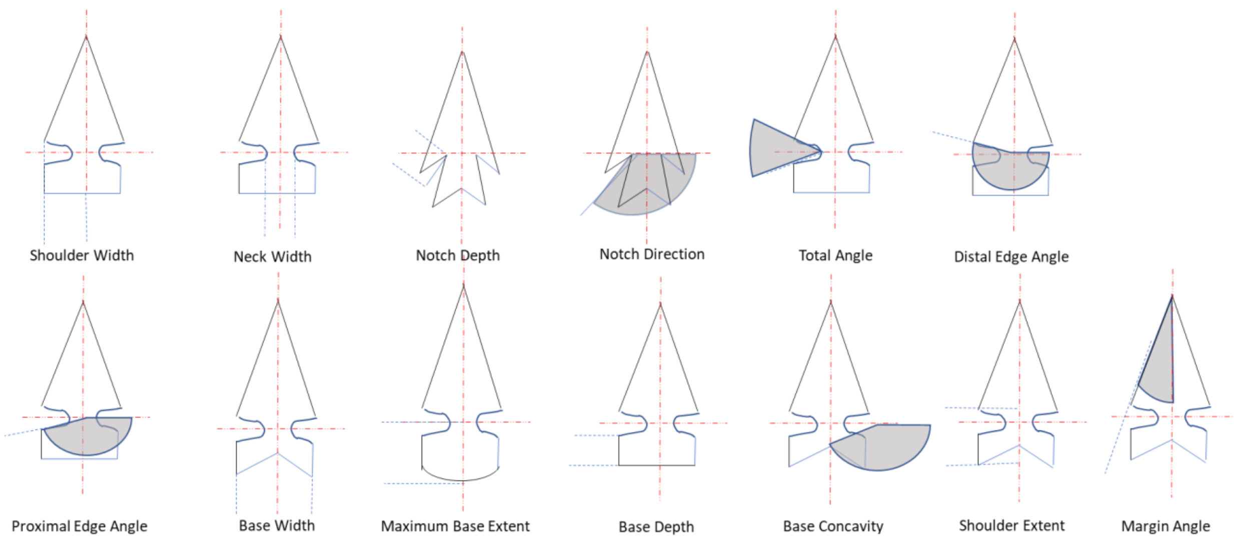

Figure 3. Definition of Projectile Point Properties

There are two types of unsupervised, objective clustering methods, those which cluster objects into the number of groups decided by the analyst and those that create clusters of objects based only on the data, i.e., values of the parameters of the objects to be grouped. In this investigation Affinity Propagation (AP) (Dueck 2009) was used to identify the number of groups using only the parameter data. That number of groups was then used in Kmean (Geron 2019) and Agglomerative Hierarchical Clustering (AHC) (Murtagh and Legendre 2014) to also identify groups for comparison between different objective clustering methods.

Comparison of subjective and objective grouping

Four analyses were done to compare the subjective and objective groupings and to determine if information derived from "crowdsourcing" subjective groupings could be used to increase the morphological similarity between groups derived from objective clustering. These are:

a. Comparison of the average number of groups derived by crowdsourcing subjective grouping and that using objective clustering.
b. Determine a measure of "sameness" using points that are grouped together by most volunteers.
c. Use the reduction in standard deviation by parameter to determine which parameters are the most influential in the subjective groupings.
d. Compare the amount of reduction in standard deviation for groups derived from subjective and objective groupings using all 20 parameters as well as only the most influential parameters.

Since projectile points placed in a group were chosen because they were perceived as similar by volunteers, or chosen as similar by mathematical analysis, there should be parameters of those points that have similar values. A measure of that similarity is the standard deviation (SD). Thus, the SD can be determined for each parameter within each group. The smaller the SD number, the more similar the values of that parameter are for all members of a group. Therefore, if the SD for each parameter of a group is compared with the SD of that parameter using the whole assemblage (259 points), those parameters with the most reduction in SD relative to the assemblage can be considered as the most important when grouping the points, i.e., the most influential.

An assumption made in this analysis is that the projectile point characteristics that influenced the subjective grouping are related to some or all the 20 parameters used in the objective clustering. It is possible that factors influencing the choices of the subjective groupings may be more complex combinations of some or all the 20 parameters or characteristics not included in the 20 parameters. However, the assumption is that those characteristics/parameters which are most related will still have more reduction in SD within groups. Therefore, to accomplish the analysis (list items a-c above) the SD of each parameter and amount of reduction compared to the whole assemblage of points was calculated for each group from each volunteer and groups resulting from the objective clustering.

Results

Subjective Groupings

The number of groups identified by the volunteers varied from 5 to 84. The minimum and maximum number of groups, the average number of groups for all volunteers, and the average for each of the three experience levels is shown in Table 4.

Table 4. Summary of Group Metrics by Experience Level

Experience	Minimum number of groups	Maximum number of groups	Average number of groups by experience level
All volunteers	5	84	23
Advanced	5	42	20
Avocational	5	46	16
Novice	11	84	30

The maximum number of volunteers that paired any two points was 32 (of possible 34). This occurred for only one pair of points (Figure 4a,b). Assuming that volunteers that break an assemblage into many groups are more sensitive to small changes, the two points and any other points which were common in the same group for the seven volunteers with 41 groups (mean + 1σ number of groups) or more were identified. Five projectile points meet this criterion (Figure 4a-e). These five points also occurred in a single group for at least 80% of the volunteers. The difference between the maximum and minimum values (maximum value – minimum value) for each parameter are provided in Table 5. It is proposed that these differences are good estimates for defining “sameness,” at least for the point style shown in Figure 4a-e. Points with differences larger than these can result in points being placed in other groups.

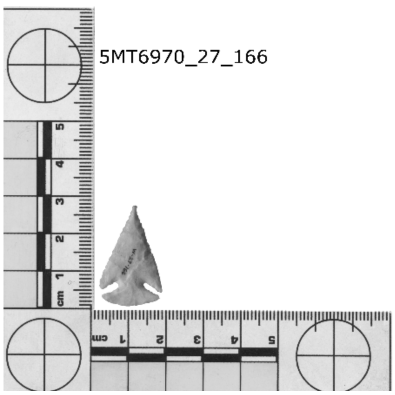

Figure 4a

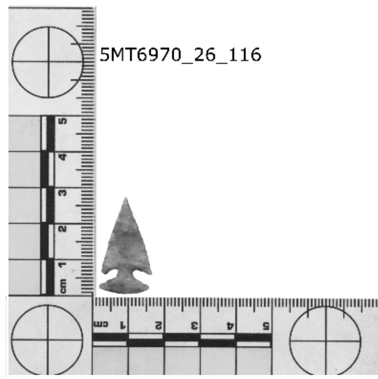

Figure 4b

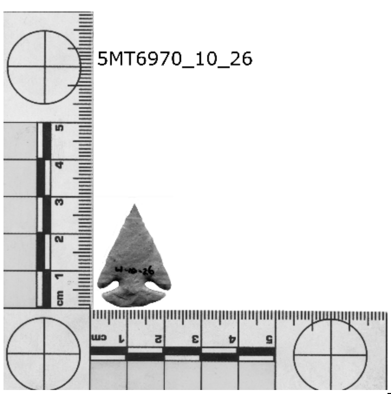

Figure 4c

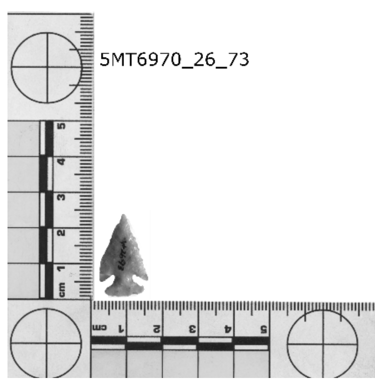

Figure 4d

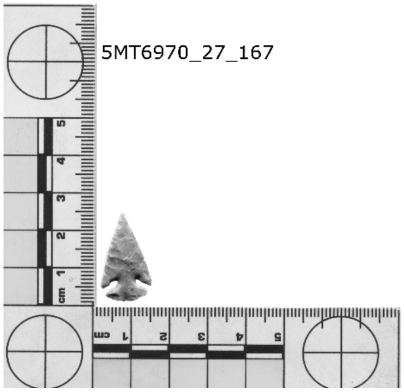

Figure 4e
Figure 4a-e. The projectile points associated within a group by at least 80% (27) of the volunteers.

Table 5: Maximum Value Difference for Co-occurring Projectile Points from Volunteers with 42 or more Groups

Shoulder Width (SW)	Neck Width (NW)	Notch Depth	Notch Direction	Total Angle	Distal Edge Angle	Proximal Edge Angle	Base Width (BW)	Maximum Base Extent (BE)	Base Depth (BD)
6.6 mm	3.1 mm	1.5 mm	20 deg	15 deg	30 deg	31 deg	4.9 mm	1.2 mm	0.9 mm

Base Concavity (BC)	Shoulder Extent (SE)	Angle of Edge	NW/SW	BW/SW	BD/BW	BE/SW	BD/SW	NW/BW	SE/BE
17 deg	1.5 mm	20 deg	0.06	0.34	0.08	0.19	0.07	0.15	0.20

When analysis using reduction in standard deviation (RSD), i.e., ratio of group standard deviation to assemblage standard deviation by parameter, is applied to the groups created by each volunteer, the parameters with the most RSD differ for each volunteer. When the combined RSD for each parameter is considered across all the volunteers some show greater reduction than others. As an example, Tables 6a-d shows the RSD by parameter for four of the volunteers that created the same number of groups, thirteen. The first set is by a career archaeologist, the next two are avocational, and the last case is a novice. Each row is the results of a group (G1, G2, and so on). Groups containing less than five points are not included.

Choosing an arbitrary, conservative amount of 30% reduction in standard deviation and counting each of those occurrences (shaded cells in the tables) for each parameter for each group in the four cases (Tables 6a-d), a measure of the influence of each parameter on the groupings can be determined. The larger the count, the more times the standard deviation of that parameter has been reduced by at least 30% when forming the groups. Table 7 shows the normalized (counts divided by number of groups with five or more members) and the totals for all four cases. The top 50% (all those above the mean of 1.408) have been shaded and are considered the most influential. From a compilation of these four cases, it appears that Base Depth, Base Depth/Shoulder Width, Base Width/Shoulder Width, Base Depth/Base Width, Proximal Edge Angle, Neck Width/Base Width, and Base Concavity have more influence than the other parameters, and that Angle of the Edge has no influence in defining the groupings by these four volunteers.

Table 6a: Reduction in Standard Deviation by Group, Case 1 (Career Archaeologist)

	Shoulder Width (SW)	Neck Width (NW)	Notch Depth	Notch Direction	Total Angle	Distal Edge Angle	Proximal Edge Angle	Base Width (BW)	Maximum Base Extent (BE)	Base Depth (BD)
G1	0.80	0.72	1.12	0.95	0.99	0.86	1.07	0.94	0.80	0.50
G2	0.96	0.72	1.02	0.42	0.30	0.48	0.42	0.79	0.60	0.17
G3	0.63	0.66	0.76	0.61	0.73	0.75	0.58	0.75	0.92	0.64
G4	0.85	0.78	0.73	0.57	0.76	0.69	0.68	0.84	0.82	0.41
G5	0.84	1.08	0.39	0.97	1.07	1.15	0.73	0.92	1.08	0.80
G6	0.61	0.75	0.50	0.71	0.56	0.79	0.73	0.72	0.57	0.37
G7	1.26	0.89	1.09	0.90	1.20	1.17	0.52	0.93	0.87	0.48
G8	0.81	0.74	0.99	0.72	1.02	1.00	0.78	0.82	0.82	0.70
Count	2	1	2	3	2	2	4	0	2	6

	Base Concavity (BC)	Shoulder Extent (SE)	Angle of Edge	NW/SW	BW/SW	BD/BW	BE/SW	BD/SW	NW/BW	SE/BE
G1	1.27	0.89	1.19	0.92	0.78	0.99	0.58	0.51	1.34	1.36
G2	0.58	0.54	0.94	0.59	0.53	0.21	0.55	0.19	0.59	0.68
G3	0.56	0.87	0.79	0.76	0.56	0.66	1.00	0.69	0.62	0.46
G4	0.72	0.69	0.93	0.74	0.58	0.48	0.57	0.49	0.50	0.70
G5	0.99	1.10	1.11	0.64	0.78	0.75	1.17	0.83	0.66	0.99
G6	0.78	0.52	1.23	0.58	0.56	0.47	0.75	0.48	0.47	0.67
G7	0.71	0.75	1.40	0.49	0.47	0.66	0.70	0.43	0.75	1.29
G8	0.83	1.01	0.74	0.86	0.62	0.73	0.63	0.45	0.86	1.33
Count	2	3	0	4	6	5	4	7	5	3

Table 6b: Reduction in Standard Deviation by Group, Case 2 (Avocational Experience)

	Shoulder Width (SW)	Neck Width (NW)	Notch Depth	Notch Direction	Total Angle	Distal Edge Angle	Proxim al Edge Angle	Base Width (BW)	Maximum Base Extent (BE)	Base Depth (BD)
G1	0.79	0.77	0.78	0.4	0.79	0.68	0.63	0.87	0.84	0.62
G2	0.66	0.78	0.53	0.71	0.72	0.89	0.66	0.79	1.00	0.70
G3	0.67	0.64	084	0.58	0.76	0.70	0.53	0.76	0.92	0.63
G4	1.22	1.04	0.72	1.01	0.64	0.94	0.68	0.90	1.04	0.62
G5	1.57	1.55	0.88	0.78	0.82	0.81	0.99	1.42	1.05	0.60
G6	1.75	1.66	0.95	1.16	1.02	1.27	0..58	1.48	1.50	0.61
G7	1.08	0.96	1.12	0.41	0.67	0.45	0.53	1.03	0.80	0.29

Count	2	1	1	3	2	2	6	0	0	6

	Base Concavity (BC)	Shoulder Extent (SE)	Angle of Edge	NW/SW	BW/SW	BD/BW	BE/SW	BD/SW	NW/BW	SE/BE
G1	0.72	0.76	1.03	0.75	0.59	0.65	0.74	0.65	0.64	0.56
G2	0.65	0.95	0.98	0.63	0.72	0.67	0.98	0.70	0.60	0.68
G3	0.56	0.82	0.63	0.79	0.53	0.65	1.00	0.70	0.59	0.41
G4	0.96	1.03	0.94	0.81	0.74	0.71	1.22	0.51	0.60	0.91
G5	1.36	0.97	1.61	0.78	0.50	1.16	0.73	0.53	1.02	1.33
G6	1.39	1.22	1.29	0.72	0.61	0.65	0.69	0.59	0.67	1.10
G7	0.88	0.82	0.90	0.76	0.66	0.39	0.52	0.29	0.96	0.95
Count	2	0	0	1	5	5	2	5	5	3

Table 6c: Reduction in Standard Deviation by Group, Case 3 (Avocational experience)

	Shoulder Width (SW)	Neck Width (NW)	Notch Depth	Notch Direction	Total Angle	Distal Edge Angle	Proximal Edge Angle	Base Width (BW)	Maximum Base Extent (BE)	Base Depth (BD)
G1	0.65	0.65	0.72	0.63	0.78	0.79	0.60	0.76	0.97	0.67
G2	0.81	0.95	0.77	0.73	0.92	0.74	0.77	1.02	0.89	0.51
G3	1.00	0.68	1.30	0.32	0.64	0.32	0.59	1.00	0.79	0.40
G4	0.77	1.21	0.82	0.77	1.18	1.08	0.55	0.71	0.84	0.61
G5	1.48	1.20	0.98	1.20	1.02	1.29	0.74	1.15	0.92	0.62
G6	0.84	1.08	0.54	1.10	0.59	1.11	0.76	1.04	0.49	0.23
G7	1.25	1.46	0.93	0.62	0.58	0.62	0.74	1.20	1.07	0.43
Count	1	2	1	3	3	2	3	0	1	7

	Base Concavity (BC)	Shoulder Extent (SE)	Angle of Edge	NW/SW	BW/SW	BD/BW	BE/SW	BD/SW	NW/BW	SE/BE
G1	0.60	0.92	0.81	0.72	0.61	0.66	0.99	0.69	0.59	0.57
G2	0.81	0.79	1.06	0.84	0.77	0.55	0.66	0.51	0.87	0.56
G3	0.83	0.60	1.05	0.77	0.81	0.73	0.70	0.40	0.99	0.80
G4	0.63	0.74	0.79	1.12	0.38	0.51	0.67	0.45	0.84	0.71
G5	1.13	1.16	1.40	0.73	0.80	0.77	1.07	0.55	0.72	1.48
G6	0.70	0.39	1.56	0.64	0.31	0.59	0.96	0.52	0.60	0.90
G7	0.60	0.96	1.00	0.76	0.64	0.41	0.41	0.33	0.76	1.05
Count	3	2	0	1	4	5	3	7	2	2

Table 6d: Reduction in Standard Deviation by Group, Case 4 (Novice Experience)

	Shoulder Width (SW)	Neck Width (NW)	Notch Depth	Notch Direction	Total Angle	Distal Edge Angle	Proximal Edge Angle	Base Width (BW)	Maximum Base Extent (BE)	Base Depth (BD)
G1	0.62	0.65	0.54	0.95	0.53	1.19	0.47	0.54	0.95	0.68
G2	1.43	1.31	0.95	1.57	0.77	0.86	0.95	0.90	1.01	0.69
G3	1.16	0.83	0.87	0.80	0.73	0.81	0.65	0.90	0.90	0.72
G4	0.55	0.72	0.72	0.76	0.77	0.77	0.68	0.79	0.90	0.71
G5	1.29	1.23	1.26	0.93	0.87	0.80	0.98	1.25	0.90	0.58
G6	0.74	0.73	0.74	0.73	1.00	0.77	0.92	1.00	0.87	0.62
G7	0.76	0.64	0.74	0.67	0.86	0.99	0.69	0.78	1.08	0.76
Count	2	2	1	1	1	0	4	1	0	4

	Base Concavity (BC)	Shoulder Extent (SE)	Angle of Edge	NW/SW	BW/SW	BD/BW	BE/SW	BD/SW	NW/BW	SE/BE
G1	0.71	0.88	1.07	0.45	0.68	0.88	1.46	0.99	0.63	0.67
G2	0.61	0.94	0,90	0.95	1.24	0.88	1.38	0.59	1.10	0.71
G3	1.03	1.01	1.02	0.49	0.61	0.78	0.74	0.61	0.77	1.07
G4	0.56	0.83	0.76	0.77	0.68	0.70	0.94	0.71	0.83	0.92
G5	1.16	0.89	1.13	1.02	1.03	0.83	0.57	0.50	1.40	1.30
G6	0.66	0.82	1.09	0.78	0.81	0.60	0.57	0.58	0.84	0.66
G7	0.67	1.04	0.86	0.75	0.68	0.69	1.11	0.72	0.51	0.82
Count	4	0	0	2	4	3	2	4	2	2

Table 7: Relative Influence of Parameters for Volunteers that Identified 13 Groups

	Shoulder Width (SW)	Neck Width (NW)	Notch Depth	Notch Direction	Total Angle	Distal Edge Angle	Proximal Edge Angle	Base Width (BW)	Maximum Base Extent (BE)	Base Depth (BD)
Case 1	0.250	0.125	0.250	0.375	0.250	0.250	0.500	0	0.250	0.750
Case 2	0.286	0.143	0.143	0.429	0.286	0.286	0.857	0	0	0.857
Case 3	0.143	0.286	0.143	0.429	0.429	0.286	0.429	0	0.143	1.000
Case 4	0.286	0.286	0.143	0.143	0.143	0	0.571	0.143	0	0.571
Total	0.965	0.840	0.679	1.376	1.108	0.822	2.357	0.143	0.393	3.178

	Base Concavity (BC)	Shoulder Extent (SE)	Angle of Edge	NW/SW	BW/SW	BD/BW	BE/SW	BD/SW	NW/BW	SE/BE
Case 1	0.250	0.375	0	0.500	0.750	0.625	0.500	0.875	0.625	0.375
Case 2	0.286	0	0	0.143	0.714	0.714	0.286	0.714	0.714	0.429
Case 3	0.429	0.286	0	0.143	0.571	0.714	0.429	1.000	0.286	0.286
Case 4	0.571	0	0	0.286	0.571	0.429	0.286	0.571	0.286	0.286
Total	1.536	0.661	0	1.072	2.606	2.482	1.501	3.160	1.911	1.376

Table 8 shows the results when this same type of analysis is done on the groups from all the volunteers and by each experience level. Then the parameters that had values larger than the mean for each experience group are considered to have the most influence and are shaded in Table 8. Using this criterion, it appears that the parameters which influence grouping the most are the same no matter what the experience level of the volunteer, except for Shoulder Width which is included in the Novice experience case and not in the others.

Using the results from all the volunteers in Table 8, there are nine parameters that are most related to the subjective criterions used by all the volunteers to group the points. They are Notch Direction, Proximal Edge Angle, Base Depth, Base Width/Shoulder Width, Base Depth/Base Width, Base Extent/Shoulder Width, Base Depth/Shoulder Width, Neck Width/Base Width, and Shoulder Extent/Maximum Base Extent. The least influential parameters are Angle of the Edge and Maximum Base Extent. Based on the results in Table 8, it appears projectile point proportions (ratios of individual characteristics) generally have more influence on the subjective grouping decisions than the individual characteristics that comprise the ratio.

Table 8: Influence of each Parameter by Experience Group and for All Volunteers
(E1=Advanced, E2= Avocational, E3=Novice)

	Shoulder Width (SW)	Neck Width (NW)	Notch Depth	Notch Direction	Total Angle	Distal Edge Angle	Proximal Edge Angle	Base Width (BW)	Maximum Base Extent (BE)	Base Depth (BD)
E1	1.92	1.73	1.88	3.32	2.03	1.96	3.38	0.89	1.04	5.45
E2	3.62	2.79	3.46	5.39	3.42	2.78	4.84	1.11	1.24	7.83
E3	5.67	4.48	5.16	6.06	5.09	3.60	7.00	3.44	2.96	7.94
All	11.21	9.00	10.50	14.77	10.54	8.34	15.22	5.44	5.24	21.22

	Base Concavity (BC)	Shoulder Extent (SE)	Angle of Edge	NW/SW	BW/SW	BD/BW	BE/SW	BD/SW	NW/BW	SE/BE
E1	2.38	1.38	0.65	2.30	4.36	4.27	3.33	5.75	2.93	3.18
E2	3.57	1.59	0.71	3.16	5.97	5.34	4.34	7.84	5.56	5.00
E3	5.26	3.33	2.95	5.13	7.31	7.14	5.92	8.18	7.29	6.63
All	11.21	6.30	4.31	10.59	17.64	16.75	13.59	21.77	15.78	14.81

<u>Objective Groupings</u>
The objective clustering algorithm AP identified 22 clusters, using the same 20 parameters, when applied to the same 259 projectile points that were given to the volunteers. When the 9 most influential parameters from the volunteer grouping were used instead, the number of clusters dropped to 20.

Clustering algorithms attempt to create clusters that have less variation of the parameter values within a cluster than between clusters. Often this is measured by the multi-dimensional Euclidean distance between the centroid and the points within the cluster, where the dimensions are the parameters. The clusters will reduce the standard deviation of the parameters within a cluster. The more highly correlated the parameters are within each cluster, the tighter the standard deviation within that cluster. The same analysis that was done on the subjective groups was applied to the groups from the objective clustering. Table 9 shows the results, with the most highly correlated parameters shaded for each clustering algorithm as well as the cumulative result using all three algorithms. Each algorithm had eleven parameters in the top 50% while the cumulative result gave ten parameters in the top 50%. The Base Depth/Shoulder Width ratio was the most correlated parameter within the clusters for all three algorithms. Distal Edge Angle and Neck Width/Base Width fell just short of the mean which was 2.283.

Table 9: Indication of Correlation of Parameters for the Objective Clustering Algorithms

	Shoulder Width (SW)	Neck Width (NW)	Notch Depth	Notch Direction	Total Angle	Distal Edge Angle	Proximal Edge Angle	Base Width (BW)	Maximum Base Extent (BE)	Base Depth (BD)
AP	0.705	0.588	0.824	0.824	0.882	0.824	0.882	0.588	0.824	0.941
AHC	0.667	0.667	0.944	0.889	0.944	0.722	0.833	0.667	0.778	1
KMEAN	0.706	0.765	1	0.765	0.882	0.706	0.647	0.470	0.882	1
All	2.078	2.020	2.768	2.478	2.708	2.252	2.362	1.725	2.484	2.941

	Base Concavity (BC)	Shoulder Extent (SE)	Angle of Edge	NW/SW	BW/SW	BD/BW	BE/SW	BD/SW	NW/BW	SE/BE
AP	0.412	0.706	0.353	0.882	0.588	0.941	0.647	1	0.824	0.647
AHC	0.444	0.667	0.500	0.944	0.667	0.944	0.833	1	0.722	0.833
KMEAN	0.588	0.824	0.176	0.882	0.647	0.941	0.824	1	0.706	0.706
All	1.444	2.197	1.029	2.708	1.902	2.826	2.304	3.000	2.252	2.186

To have a meaningful typology, clusters or groups should be composed of projectile points that look more like each other than to projectile points in other groups. That means that the individual measurements and proportions of the points within a group should be more similar, i.e., have a smaller standard deviation, than the relative reduction of standard deviation of the whole assemblage of points. For this to be true, one way to compare the results of objective clustering to subjective grouping is to compare the amount that the standard deviation of the parameters has been reduced relative to the assemblage because of clustering or grouping.

Using the nine parameters that appear to be most influential in the volunteer groupings in the three clustering algorithms Affinity Propagation clustering (AP), Agglomerative Hierarchical Clustering (AHC), and KMEAN clustering (KM), an average relative reduction in the standard deviation was obtained for each algorithm. This is compared to the average relative reduction in standard deviation for all the volunteer groupings and for the groupings by experience level (Table 10).

Table 10. Comparison of Average Reduction in Standard Deviation using the Nine Most Influential Subjective Parameters

Method of Grouping	Relative Reduction in Standard Deviation
AP (20 clusters)	0.49 (51% reduction)

AHC (20 clusters)	0.45 (55% reduction)
KM (20 clusters)	0.45 (55% reduction)
All Volunteers	0.74 (26% reduction)
Advanced	0.73 (27% reduction)
Avocational	0.77 (23% reduction)
Novice	0.72 (28% reduction)

Table 11 gives the reduction in standard deviation using all 20 parameters. The objective algorithms reduce the standard deviation by almost twice as much as the subjective groupings. All the objective methods provide approximately the same amount of reduction. Similarly, the amount of experience appears to have little effect on the reduction resulting from the subjective groupings. For comparison with the objective and subjective results, the average reduction in standard deviation from 10 random cases was determined and is shown in Table 11.

To obtain the random selection, a group number between 1 and 22 was selected at random. Then a projectile point was randomly selected from the 259 points for that group. This was repeated until all projectile points had been selected and placed in a group. The reduction in standard deviation analysis was then applied to those groups. This was repeated 10 times and an average reduction in standard deviation was obtained. The 4% reduction is the result of a small number of random cases being used. Aa more and more cases are used, the reduction should approach 0% reduction in standard deviation.

Table 11. Comparison of Average Reduction in Standard Deviation using all Twenty Parameters

Method of Grouping	Relative Reduction in Standard Deviation
AP (22 clusters)	0.57(43% reduction)
AHC (22 clusters)	0.57 (43% reduction)
KM (22 clusters)	0.58 (42% reduction)
All Volunteers	0.82 (18% reduction)
Advanced	0.82 (18% reduction)
Avocational	0.85 (15% reduction)
Novice	0.79 (21% reduction)
Random Selection	0.96(4% reduction)

Discussion

Even though thirty-four volunteers are a small number of responses from which to obtain the “knowledge of the crowd” from crowdsourcing, the average number of groups from crowdsourcing was close to that obtained from the objective AP clustering algorithm. It is also obvious from the results there is a wide range in the number of groups that the volunteers identified in the same assemblage, even within the three experience groups.

The projectile points that occurred together more often in the subjective grouping were those that were the most different from the others, i.e., less subject to where boundaries between groups were, and yet like each other. The assumption is that those volunteers that had larger numbers of groups are more sensitive to small differences, but criterions for grouping are likely person specific. Given that, the difference between the largest and smallest values of each parameter can be considered the smallest difference that is recognizable by the volunteers. These parametric differences might be style dependent and different for people outside the cultures that made and used points. The former can be tested, and the latter may be informed by other study designs but is likely unknowable.

Analysis of crowdsourcing groups with respect to metric parameters and proportions does appear to inform which of those parameters are most related to characteristics of the projectile points that are important or influential when subjectively grouping. It is interesting that those influential parameters are nearly identical and no dependent upon the experience level of the person defining a grouping. This does not mean the groups from persons in the same or different experience levels will necessarily put the same projectile points within a group. The relative importance of the parameters, other characteristics not captured in the metrics and proportions, and decisions on where the boundaries are that distinguish one group from another are likely factors not captured in this analysis.

The reduction in standard deviation is an indication of how similar the values of each parameter are for points in a group. When these values are compared for groups derived by subjective and objective methods, the groups derived from the objective methods have much greater (nearly twice) reduction in standard deviations. This indicates the points within each group created using the objective algorithms are more similar. Objective methods also produce repeatable results, given the same algorithm and dataset. While measuring subjective repeatability was not tested or designed into this study, there were some anecdotal indications that subjective grouping may not be entirely repeatable.

Conclusions

This exploratory investigation indicates that crowdsourcing may provide insight into the parameters that should be used in objective clustering to identify and define styles or a typology. The results in this paper should be considered exploratory and the specifics such as the most influential parameters may be different for different mixes of projectile point types. It also appears that nuances that are likely used in subjective grouping may not be entirely represented by the objective measurements and proportions used in this study.

Due to an oversight two images of the same point were included in some image sets provided to volunteers. Those images were often put in different groups, i.e., raising the issue of drift within a group as well as repeatability of assignment to a group. A future crowdsource investigation should build on these results as well as address the weaknesses of this study. A more balanced set of projectile points shapes, many more respondents, and more projectile point parameters might provide more generally applicable results.

The comparison of the subjective and objective approaches to grouping or clustering indicate that objective approaches do provide groups that contain points that are more like each other, based on projectile point characteristics, than subjective clustering. In addition, using the same objective algorithm on the same assemblage does give repeatable results. These results do indicate a path to a more repeatable, standardized approach to create typologies that can be applied to a variety of spatial and temporal scales.

Finally, the question remains as to how well the influential parameters that result from crowdsourcing analysis such as shown in this study inform the preferences, training, and social influences of the people that made the projectile points.

Acknowledgments

I'd like to thank Matt Landt, Teri Hauser, and Craig Banister for reviewing earlier versions of this paper. Their helpful comments and suggestions resulted in a much better paper.

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